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Centre Number

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Student Number

2026 ORIGINAL HSC-STYLE MOCK

Mathematics Extension 1

Answers, worked solutions and marking guidance

Teacher edition

70 marks. Binding scope: 2017 Mathematics Extension 1 for HSC 2026, including Year 11 Extension 1 and Mathematics Advanced assumed knowledge. These are original practice questions; neither historical frequency nor a close visual resemblance establishes a reliable prediction.

Design rationale

The 2020-2025 official examinations are the primary same-syllabus pattern evidence. Detailed recent analysis records 75 written items from 2023-2025, with original visual questions checked. The local 2024-2026 trial library supplies secondary variants, not authority to exceed the syllabus. The paper progresses from routine skills to reciprocal graph construction, probability interpretation, washer geometry, and a reverse-inference projectile capstone.

Format and balance

Section I has 10 one-mark items. Section II has 25 separately marked items, worth 15, 14, 16 and 15 marks across Questions 11-14. The largest single item is 4 marks. Statistics occupies 6/70 marks, not the personalised 23-mark weighting in the previous three-paper pack. Standard-topic balance is not a forecast of 2026 topic weights.

Use

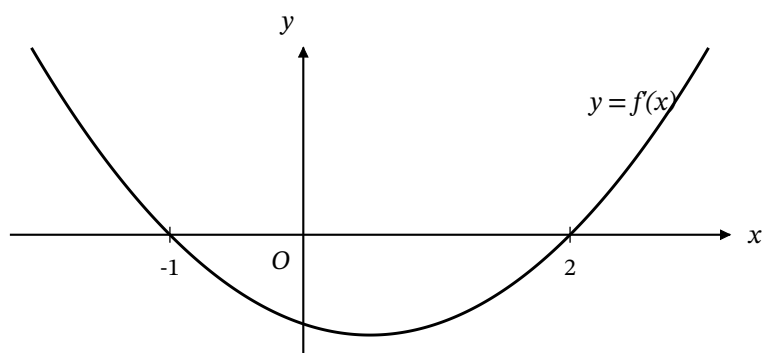
Accept mathematically equivalent methods and justified error-carried-forward reasoning. Marking points are proposed teacher guidance, one listed point per mark, not official NESA criteria. Estimated workload is about 15 minutes for MCQ and 100-105 minutes for written work; this has not been student-piloted.

Section I - answer key and explanations

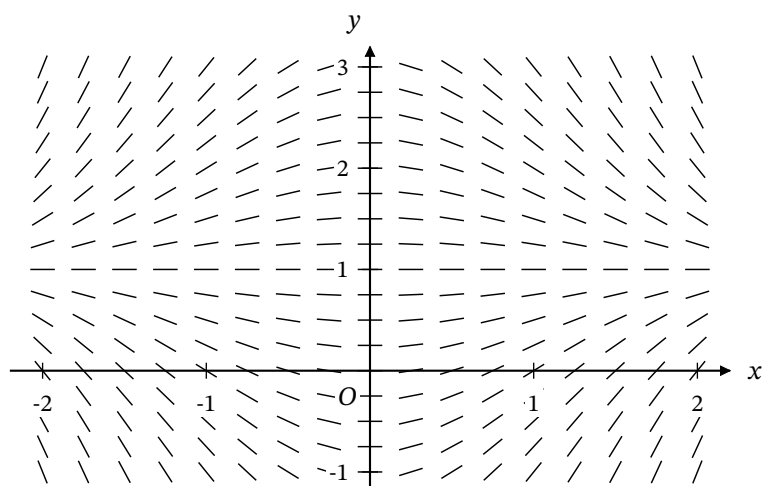
Question	1	2	3	4	5	6	7	8	9	10
Answer	B	D	A	C	B	D	B	C	C	D

1. B Differentiating a factor of multiplicity three reduces its multiplicity to two: $P'(x) = (x - 2)^2(x + 1)(5x - 1)$
2. D $\mathbf{a} \cdot \mathbf{b} = -1$ and $|\mathbf{b}|^2 = 2$, so the projection is $-\frac{1}{2}\mathbf{b}$.
3. A $du = e^x dx$, and the endpoints become 1 and 2.
4. C At $x = -1$, f' changes from positive to negative. At 2 it changes from negative to positive, giving a minimum.
5. B $1 - p = \text{Var}(X)/E(X) = 2/3$, so $p = 1/3$ and $n = 9$.
6. D The principal range of arccos is $[0, \pi]$, and $\cos(7\pi/6) = \cos(5\pi/6)$.
7. B $\sin 3x - \sin x = 2 \sin x \cos 2x$. The solutions are $0, \pi/4, 3\pi/4, \pi$.
8. C The slopes are zero on both $x = 0$ and $y = 1$. For $x > 0$, slopes are positive below $y = 1$ and negative above it. Only C has all these properties.
9. C The general term has power $12 - 3r$, so $r = 4$. The coefficient is $\binom{6}{4} 2^4 = 240$.
10. D Write $g = f^{-1}$. Then $h'(x) = 2xg'(x^2 + 1)$, so $h'(0) = 2g'(1) = 2/f'(0) = 1$.

Visual MCQ checks



Q4: positive-to-negative slope sign at -1 gives the local maximum of f . The plotted curve is f' , not f .



Q8: zero slope on $x=0$ and $y=1$; opposite signs across both sets. A few test points distinguish the options.

Quick answers - Question 11

(a) $2/3 \leq x \leq 4$.

(b) $y' = \frac{1}{2(x+1)\sqrt{2x+1}}$, for $x > -1/2$.

(c) $1/12$.

(d) Endpoints $(0, \pi/2), (2, -\pi/2)$; x-intercept $(1, 0)$.

(e) 88.

(f) $\mathbf{a} = (3/10, -1/10) + (7/10, 21/10)$.

(g) $\pi/6 + 2 - \sqrt{3}$.

Quick answers - Question 12

(a)(i) $\mathbb{R} \setminus \{-2, 2\}$.

(a)(ii) Vertical asymptotes $x = \pm 2$; horizontal asymptote $y = 1$; y-intercept $(0, -1/4)$; no x-intercepts.

(b) The identity holds for all positive integers.

(c) $\frac{dh}{dt} = \frac{1}{4\pi} \text{ cm s}^{-1}$.

(d) 480.

(e) $x = \pi/2, 7\pi/6, 11\pi/6$.

Quick answers - Question 13

(a)(i) 0.02.

(a)(ii) $P(\hat{p} < 0.37 \text{ or } \hat{p} > 0.43) \approx 0.1336$.

(a)(iii) 1067.

(b)(i) $4/3$ square units.

(b)(ii) $104\pi/15$ cubic units.

(c)(i) $\ln \frac{3+y}{3-y} = 3t^2 + 6t$.

(c)(ii) $t = \sqrt{1 + \frac{\ln 3}{3}} - 1$; limiting value 3.

Quick answers - Question 14

(a)(i) $\overrightarrow{OC} = \frac{\mathbf{a} + 2\mathbf{b}}{3}$.

(a)(ii) $\sqrt{35}$ square units.

(b)(i) $(3\sqrt{2}, 11\sqrt{2}) \text{ m s}^{-1}$.

(b)(ii) $T = 11\sqrt{2}/5 \text{ s}$; the stated expression holds on $[0, T]$.

(b)(iii) $13\sqrt{2}/10 < t < 2\sqrt{2}$; P gives a strict local minimum.

Question 11 - worked solutions

(a) (2 marks)

The right side must be nonnegative. The equivalent inequalities $-x - 1 \leq 2x - 3 \leq x + 1$ give $x \geq 2/3$ and $x \leq 4$. On this interval $x + 1 > 0$, so no extra candidates remain.

Marking: 1. Correct handling of the absolute value and sign condition; 2. Both endpoints with the correct closed interval.

(b) (2 marks)

The chain rule gives $y' = \{1 + (2x + 1)\}^{-1}/\sqrt{2x + 1}$. This simplifies to the stated expression. At $x = -1/2$, the function exists but its derivative is not finite.

Marking: 1. Correct chain-rule derivative; 2. Correct simplification and open derivative interval.

(c) (2 marks)

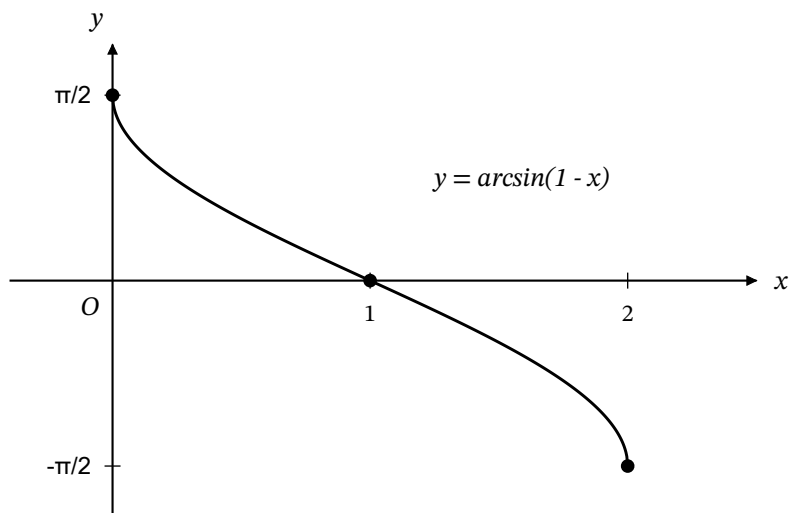
Use $\sin x \cos 2x = \frac{1}{2}(\sin 3x - \sin x)$. Hence the integral is $[-\cos 3x/6 + \cos x/2]_0^{\pi/3} = 5/12 - 1/3 = 1/12$.

Marking: 1. Correct product-to-sum conversion or equivalent valid method; 2. Correct primitive and exact endpoint evaluation.

Question 11 - worked solutions

(d) (2 marks)

The argument condition is $-1 \leq 1 - x \leq 1$, so $0 \leq x \leq 2$. The curve decreases from $(0, \pi/2)$, through $(1, 0)$, to $(2, -\pi/2)$, with closed endpoints and vertical endpoint tangents.



Marking: 1. Correct decreasing inverse-sine shape and closed endpoints; 2. Correct labelled endpoints and x-intercept.

(e) (2 marks)

Vieta gives $\alpha + \beta + \gamma = 3$, $\alpha\beta + \beta\gamma + \gamma\alpha = -4$, and $\alpha\beta\gamma = -12$. Therefore the requested value is $(-4)^2 - 2(-12)(3) = 88$. All roots are real: the polynomial factors as $(x - 3)(x - 2)(x + 2)$.

Marking: 1. Correct symmetric relations including coefficient signs; 2. Correct squared-relation expansion and result.

(f) (2 marks)

The parallel component is $(\mathbf{a} \cdot \mathbf{b})\mathbf{b}/|\mathbf{b}|^2 = (3, -1)/10$. Subtracting it from $\mathbf{a}$ gives $(7, 21)/10$, whose dot product with $\mathbf{b}$ is zero.

Marking: 1. Correct parallel projection; 2. Correct perpendicular remainder.

Question 11 - worked solutions

(g) (3 marks)

Since $du = dx/x$, the bounds become 0 and 1. The integral is $\int_0^1 (1+u)/\sqrt{4-u^2} du$. A primitive is $\arcsin(u/2) - \sqrt{4-u^2}$. Its endpoint difference is $\pi/6 - \sqrt{3} + 2$.

Marking: 1. Correct substitution and transformed bounds; 2. Correct two-term primitive; 3. Correct exact evaluation.

Question 12 - worked solutions

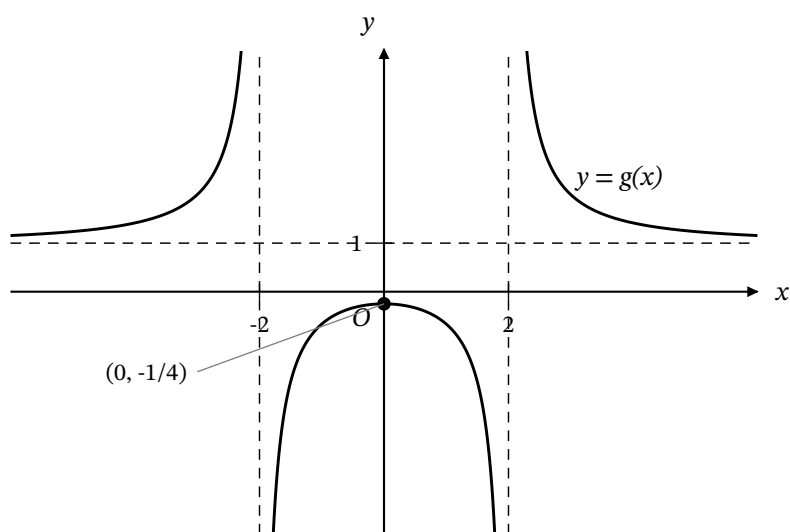
(a)(i) (1 mark)

The supplied function is defined for all real x and is zero exactly at $-2, 2$. These are the two excluded inputs of its reciprocal.

Marking: 1. Both and only excluded inputs.

(a)(ii) (3 marks)

The graph is even. On $(-2, 2)$, it is negative and reaches its maximum $-1/4$ at $x=0$; it tends to $-\infty$ on approaching either excluded endpoint from inside. On each outside interval it is above 1, tends to $+\infty$ at the nearby pole, and approaches 1 from above at infinity.



Marking: 1. Correct vertical and horizontal asymptotes; 2. Correct y-intercept and no zeros; 3. All three branches with correct one-sided behaviour and symmetry.

Question 12 - worked solutions

(b) (3 marks)

The base case is $1/3 = 1/3$. Assume the formula at $n=m$. Adding the next term gives

$\frac{m}{2m+1} + \frac{1}{(2m+1)(2m+3)} = \frac{2m^2+3m+1}{(2m+1)(2m+3)} = \frac{m+1}{2m+3}$, the required $n=m+1$ expression. Conclude by induction.

Marking: 1. Correct base case; 2. Explicit induction hypothesis and addition of the correct next term; 3. Correct factorisation, next formula and induction conclusion.

(c) (2 marks)

Similar triangles give $r = h/4$, hence $V = \pi h^3/48$ and $dV/dt = (\pi h^2/16)dh/dt$. At $h=8$, the net inflow is $2 - 8/8 = 1$. Therefore $1 = 4\pi dh/dt$.

Marking: 1. Correct one-variable volume and chain-rule rate; 2. Correct net flow, rate and units.

Question 12 - worked solutions

(d) (3 marks)

Fix A to remove rotational duplication. C is forced into the second seat clockwise from A. Of the remaining seats, B cannot use the two neighbours of A, leaving four choices. The other five people can be arranged in $5!$ ways. The total is $4 \times 5! = 480$.

Marking: 1. Correct removal of rotational symmetry and forced C position; 2. Correct four admissible seats for B; 3. Correct arrangement of remaining five people and total.

(e) (2 marks)

Using $\cos^2 x = 1 - \sin^2 x$ gives $(2 \sin x + 1)(\sin x - 1) = 0$. Thus $\sin x = 1$ or $-1/2$, giving precisely the three stated values.

Marking: 1. Correct factorisation into sine values; 2. All and only three admissible angles.

Question 13 - worked solutions

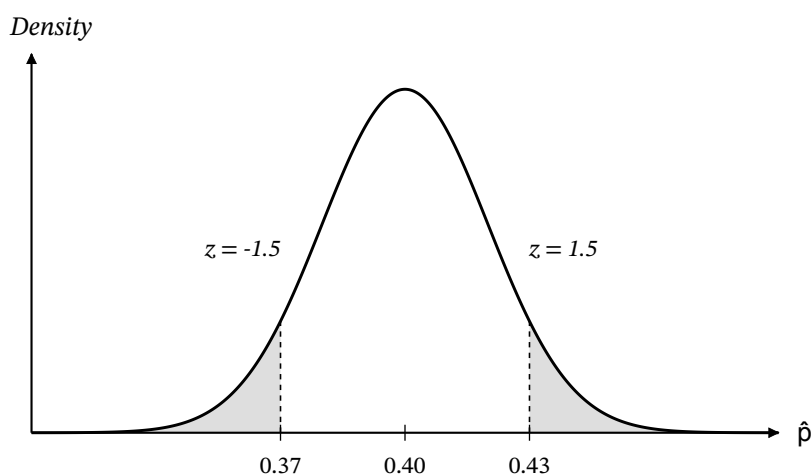
(a)(i) (1 mark)

$$SD(\hat{p}) = \sqrt{0.4(0.6)/600} = 0.02.$$

Marking: 1. Correct proportion standard deviation.

(a)(ii) (2 marks)

The two shaded tails give the stated event. Equivalent non-strict endpoint descriptions are accepted for this continuous approximation. Standardising the bounds gives $-1.5, 1.5$. Thus the probability is $2[1 - \Phi(1.5)] = 2(1 - 0.9332) = 0.1336$. The expectations 240 successes and 360 failures support the normal approximation; this is not an exact binomial probability.



Marking: 1. Correct event and both standardised bounds; 2. Correct two-tail probability using the table.

(a)(iii) (2 marks)

$\sqrt{0.24/n} \leq 0.015$ gives $n \geq 0.24/(0.015)^2 = 1066\frac{2}{3}$. The least integer is 1067. This controls standard deviation; it is not a claimed confidence guarantee.

Marking: 1. Correct squared inequality and numerical bound; 2. Correct least integer and rounding direction.

Question 13 - worked solutions

(b)(i) (2 marks)

The curves intersect at $x=1$ and $x=5$. The radical is above the line between them. Hence

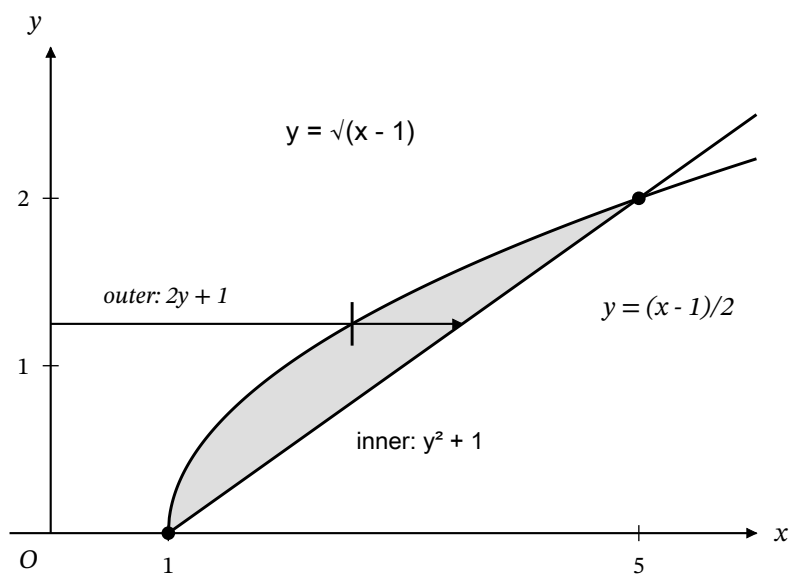
$$A = \int_1^5 [\sqrt{x-1} - (x-1)/2] dx = [\frac{2}{3}(x-1)^{3/2} - (x-1)^2/4]_1^5 = 4/3.$$

Marking: 1. Correct intersections, ordering and integral; 2. Correct exact area.

(b)(ii) (3 marks)

Write the boundaries as $x = y^2 + 1$ and $x = 2y + 1$, with $0 \leq y \leq 2$. A horizontal washer has outer radius $2y + 1$ and inner radius $y^2 + 1$. Thus

$$V = \pi \int_0^2 [(2y+1)^2 - (y^2+1)^2] dy = \pi \int_0^2 (-y^4 + 2y^2 + 4y) dy = 104\pi/15.$$



Marking: 1. Correct y-bounds and radii measured from the axis; 2. Correct washer integral with squared-radius subtraction; 3. Correct exact volume.

Question 13 - worked solutions

(c)(i) (3 marks)

Separate to obtain $dy/(9 - y^2) = (t + 1)dt$. The supplied decomposition integrates to $\frac{1}{6}[\ln(3 + y) - \ln(3 - y)] = t^2/2 + t + C$. The initial condition gives $C=0$. Multiply by 6 and combine the logarithms; both arguments are positive on the supplied y -range.

Marking: 1. Correct separation and integration with the supplied decomposition; 2. Correct use of the initial condition; 3. Correct logarithmic relation and sign/range justification.

(c)(ii) (3 marks)

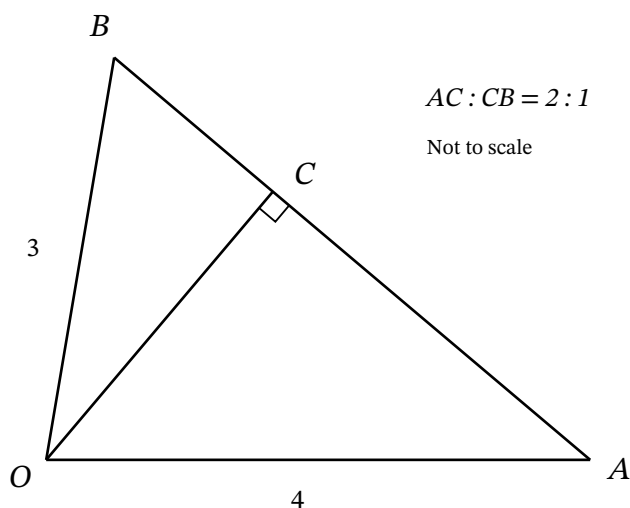
At $y=3/2$, the logarithmic ratio is $\ln 3$, so $3t^2 + 6t = \ln 3$. The nonnegative root is the stated value. Writing $E = e^{3t^2+6t}$ gives $y = 3(E - 1)/(E + 1)$. It increases from 0 towards 3 and remains strictly below 3 for finite t , so the crossing is unique.

Marking: 1. Correct threshold equation and quadratic solution; 2. Correct admissible exact time with uniqueness or monotonicity; 3. Correct limiting value from the implicit or explicit solution.

Question 14 - worked solutions

(a)(i) (2 marks)

$\vec{AC} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$, so adding $\vec{OA}$ gives $\vec{OC} = \mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a}) = (\mathbf{a} + 2\mathbf{b})/3$.



Marking: 1. Correct internal ratio and directed vector; 2. Correct position vector.

(a)(ii) (3 marks)

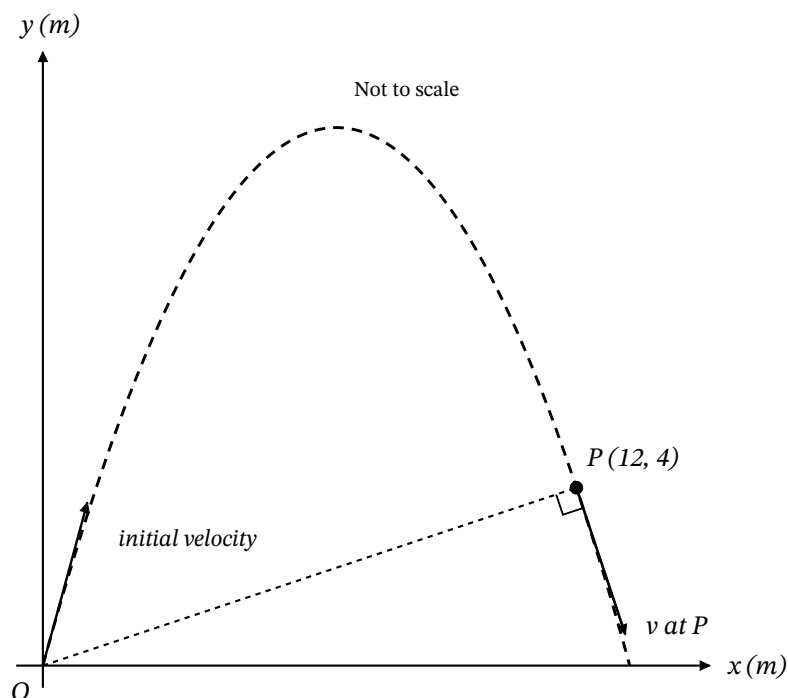
The marked right angle means $\vec{OC} \cdot (\mathbf{b} - \mathbf{a}) = 0$. Hence $(\mathbf{a} + 2\mathbf{b}) \cdot (\mathbf{b} - \mathbf{a}) = 2 - \mathbf{a} \cdot \mathbf{b} = 0$, so $\mathbf{a} \cdot \mathbf{b} = 2$. If $\theta = \angle AOB$, then $\cos \theta = 2/(4 \cdot 3) = 1/6$. Thus the area is $\frac{1}{2}(4)(3)\sqrt{1 - 1/36} = \sqrt{35}$.

Marking: 1. Correct orthogonality equation giving the scalar product; 2. Correct sine or equivalent altitude/area relation; 3. Correct positive exact area.

Question 14 - worked solutions

(b)(i) (4 marks)

Let the initial velocity be (u, v) and the time at P be $\tau > 0$. Then $u\tau = 12$, $v\tau - 5\tau^2 = 4$, and the velocity at P is $(u, v - 10\tau)$. Orthogonality gives $12u + 4(v - 10\tau) = 0$. Substitute $u = 12/\tau$, $v = 4/\tau + 5\tau$: $160/\tau - 20\tau = 0$, so $\tau = 2\sqrt{2}$. The two components are therefore $3\sqrt{2}, 11\sqrt{2}$, uniquely with $\tau > 0$.



Marking: 1. Correct component equations of motion; 2. Correct perpendicular-velocity scalar-product condition; 3. Correct positive time at P; 4. Correct unique initial velocity with units.

Question 14 - worked solutions

(b)(ii) (3 marks)

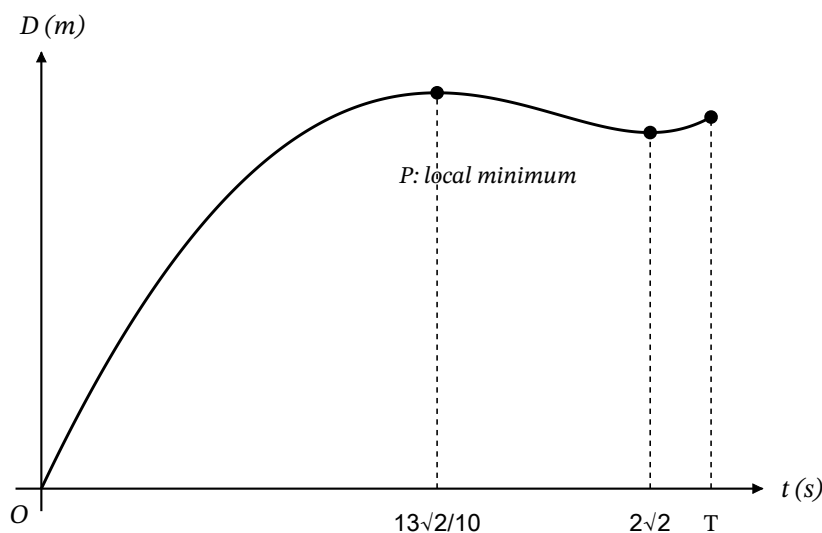
The position vector is $(3\sqrt{2}t, 11\sqrt{2}t - 5t^2)$. The positive root of $y=0$ is $T = 11\sqrt{2}/5$. Squaring and adding the components gives $18t^2 + 242t^2 - 110\sqrt{2}t^3 + 25t^4$, as required.

Marking: 1. Correct position vector and nonzero return time; 2. Correct squared-distance expansion; 3. Correct coefficients and physical time interval.

(b)(iii) (3 marks)

For $0 < t < T$, D is positive and its derivative has the sign of $(D^2)t$. Differentiation gives $10t(10t^2 - 33\sqrt{2}t + 52) = 100t(t - 13\sqrt{2}/10)(t - 2\sqrt{2})$. Both positive roots lie below T . The sign is negative only between them, and changes from negative to positive at $t = 2\sqrt{2}$, the time at P . Thus P is a strict local minimum of distance, not a maximum; it is not the global minimum, since $D(0)=0$.

Marking: 1. Correct derivative or equivalent radial-velocity sign criterion; 2. Correct two roots and decreasing interval within the flight; 3. Correct local-minimum classification with sign-change justification.



The radial maximum is not the highest point of the trajectory. At P the distance is a local minimum, while $D(0)=0$ is smaller.